<u>Lecture 22 - Dec 10</u>

<u>Graphs</u>

Partition, Cluster, Cut
Kruskal's Algorithm: Cut Property
Kruskal's Algorithm: Time Complexity

MST Problem: Partition, Cluster, Cut*

a set, whose each member is a set of vertices

a set of vertices = a piece/member of some partition.

~~a set of members of CCs~~

Iteration	Min Edge	Processing	Resulting Partition	T: MST Under Construction
Init.	—	—	$\{A\},\{B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\varnothing$
1	$w(A,B)=1$	$\because C(A) \neq C(B) \therefore$ Tree Edge	$\{A,B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\{\ (A,B)\ \}$
2	$w(B,C)=2$	$\because C(B) \neq C(C) \therefore$ Tree Edge	$\{A,B,C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\{\ (A,B),(B,C)\ \}$
3	$w(A,D)=3$	$\because C(A) \neq C(D) \therefore$ Tree Edge	$\{A,B,C,D\},\{E\},\{F\},\{G\},\{H\}$	$\{\ (A,B),(B,C),(A,D)\ \}$
4	$w(C,D)=3$	$\because C(C) = C(D) \therefore$ Internal Edge	No Change	
5	$w(E,F)=4$	$\because C(E) \neq C(F) \therefore$ Tree Edge	$\{A,B,C,D\},\{E,F\},\{G\},\{H\}$	$\{\ (A,B),(B,C),(A,D),(E,F)\ \}$
6	$w(D,E)=5$	$\because C(D) \neq C(E) \therefore$ Tree Edge	$\{A,B,C,D,E,F\},\{G\},\{H\}$	$\{\ (A,B),(B,C),(A,D),(E,F),(D,E)\ \}$
7	$w(C,F)=6$	$\because C(C) = C(F) \therefore$ Internal Edge	No Change	
8	$w(F,G)=7$	$\because C(F) \neq C(G) \therefore$ Tree Edge	$\{A,B,C,D,E,F,G\},\{H\}$	$\{\ (A,B),(B,C),(A,D),(E,F),(D,E),(F,G)\ \}$
9	$w(E,H)=8$	$\because C(E) \neq C(H) \therefore$ Tree Edge	$\{A,B,C,D,E,F,G,H\}$	$\{\ (A,B),(B,C),(A,D),(E,F),(D,E),(F,G),(E,H)\ \}$

initial partition: each vertex in its own cluster

C(A)

$\bigcup$ (Init. partition) $= V$

not a cut

C(B)

$V \setminus C(B)$

C(B) = C(D) → same C.C.

C(F) ≠ C(G) → diff. C.C.s

partition & cut.

$\bigcup$ (final partition) $= V$

final partition. all vertices in a single cluster

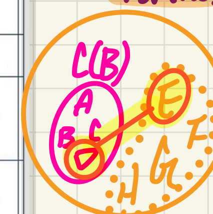
1 member

1 CC

Edge (D,E) crosses the cut.

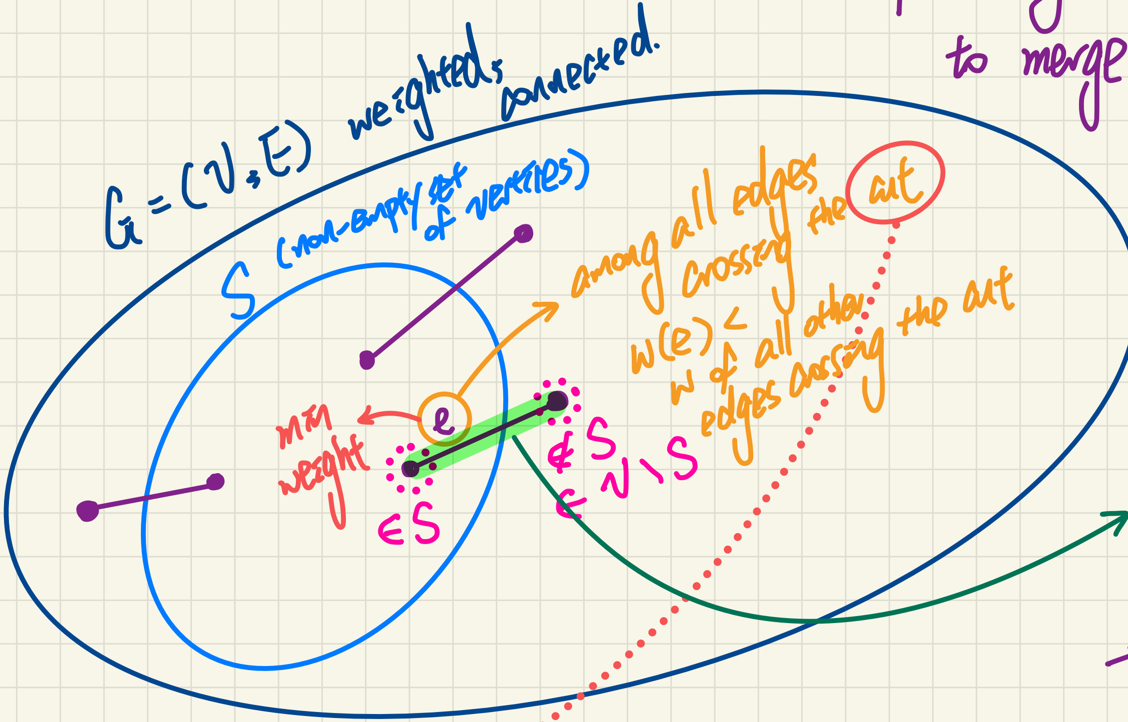
* a special case of partition: two member sets of vertices

1. 2 clusters
2. 1 cluster vs. rest of vertices



$V \setminus C(B)$

MST Problem: Cut Property



cut: $\{ S, V \setminus S \}$

member sets of partition

Kruskal's algo.

keep choosing the min edge to merge clusters, it's safe to assume that eventually a MST will be obtained.

there's an MST T s.t. $e \in T$

→ e is a safe edge

MST Problem: Cut Property in Kruskal's Algorithm

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ALGORITHM: Find-MST-Kruskal
  INPUT: Simple, Undirected, Weighted, Connected G = (V, E)
  OUTPUT: A minimum spanning tree T of G
PROCEDURE:
  for v ∈ V: C(v) := {v} -- build |V| elementary clusters
  Initialize a priority queue Q containing E -- keyed by weights
  T := ∅
  while |T| ≠ n − 1:
    (u, v) := Q.removeMin()
    let C(u) be the cluster containing u
    let C(v) be the cluster containing v
    if C(u) ≠ C(v) then
      T := T ∪ {(u, v)}
      Merge C(u) and C(v) into one cluster
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(u,v): min-weight edge crossing the cut

non-decreasing order

Iteration	Min Edge	Processing	Resulting Partition	T: MST Under Construction
Init.	—	—	$\{A\},\{B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\varnothing$
1	$w(A,B) = 1$	$\because C(A) \neq C(B) \therefore$ Tree Edge	$\{A,B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\{(A,B)\}$
2	$w(B,C) = 2$	$\because C(B) \neq C(C) \therefore$ Tree Edge	$\{A,B,C\},\{D\},\{E\},\{F\},\{G\},\{H\}$	$\{(A,B),(B,C)\}$
3	$w(A,D) = 3$	$\because C(A) \neq C(D) \therefore$ Tree Edge	$\{A,B,C,D\},\{E\},\{F\},\{G\},\{H\}$	$\{(A,B),(B,C),(A,D)\}$
4	$w(C,D) = 3$	$\because C(C) = C(D) \therefore$ Internal Edge	No Change	
5	$w(E,F) = 4$	$\because C(E) \neq C(F) \therefore$ Tree Edge	$\{A,B,C,D\},\{E,F\},\{G\},\{H\}$	$\{(A,B),(B,C),(A,D),(E,F)\}$
6	$w(D,E) = 5$	$\because C(D) \neq C(E) \therefore$ Tree Edge	$\{A,B,C,D,E,F\},\{G\},\{H\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E)\}$
7	$w(C,F) = 6$	$\because C(C) = C(F) \therefore$ Internal Edge	No Change	
8	$w(F,G) = 7$	$\because C(F) \neq C(G) \therefore$ Tree Edge	$\{A,B,C,D,E,F,G\},\{H\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G)\}$
9	$w(E,H) = 8$	$\because C(E) \neq C(H) \therefore$ Tree Edge	$\{A,B,C,D,E,F,G,H\}$	$\{(A,B),(B,C),(A,D),(E,F),(D,E),(F,G),(E,H)\}$

$\neq$ means cross

merge

merge

In the beginning of It. 6

$C(D) = \{A, B, C, D\}$

$\hookrightarrow C(D) \cap V \setminus C(D) = \varnothing$

$C(E) \subseteq V \setminus C(D)$

all edges with an end point $\in C(D)$

$(A,B), (B,C), (A,D), (C,D)$

* Merging $C(A)$ & $C(B)$ $\because$ (A,B) crosses cut $\{\underbrace{\{A\}}_{C(A)}, \underbrace{V\setminus\{A\}}_{C(B) \subseteq}\}$

** Merging $C(B)$ & $C(C)$ $\because$ (B,C) crosses cut $\{\underbrace{\{A,B\}}_{C(B)}, \underbrace{V\setminus\{A,B\}}_{C(C) \subseteq}\}$

safe to include (D,E) in T $\because$ cut property.

no cut: $\because V \setminus V = \varnothing$